

Dark energy, exotic matter and properties of horizons in black hole physics and cosmology

K.A. Bronnikov¹

*Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow 119361, Russia;
Institute of Gravitation and Cosmology, PFUR, 6 Miklukho-Maklaya St., Moscow 117198, Russia*

E. Elizalde²

Consejo Superior de Investigaciones Científicas ICE/CSIC-IEEC Campus UAB, Facultat de Ciències, Torre C5-Parell-2a pl, E-08193 Bellaterra (Barcelona) Spain

O.B. Zaslavskii³

Astronomical Institute of Kharkov V.N. Karazin National University, 35 Sumska St., Kharkov, 61022, Ukraine

*Dedicated to Sergei D. Odintsov
on the occasion of his 50th birthday*

Abstract

We summarize recent results on the properties of near-horizon metrics in different spherically symmetric space-times, including Kantowski-Sachs cosmological models whose evolution begins with a horizon (the so-called Null Big Bang) and static metrics related to black holes. We describe the types of matter compatible with cosmological and black hole horizons. It turns out, in particular, that a black hole horizon can be in equilibrium with a fluid of disordered cosmic strings (“black holes can have curly hair”). We also discuss different kinds of horizons from the viewpoint of the behavior of tidal forces acting on an extended body and recently classified as “usual”, “naked” and “truly naked” ones; in the latter case, tidal forces are infinite in a freely falling reference frame. It is shown that all truly naked horizons, as well as many of those previously characterized as naked and even usual ones, do not admit an extension and therefore must be considered as singularities. The whole analysis is performed locally (in a neighborhood of a candidate horizon) in a model-independent manner. Finally, the possible importance of some of these models in generating dynamic, perturbatively small vacuum fluctuation contributions to the cosmological constant (within a cosmological Casimir-effect approach to this problem) is discussed too.

PACS numbers: 04.70.Bw, 04.20.Dw

1 Introduction

The remarkable discovery that our Universe is accelerating [1] and its explanation, in the framework of general relativity, in terms of the so-called dark energy, have posed a number of questions. The distinctive feature of dark energy, irrespective of its specific nature, consists in the violation of the standard energy conditions, including the Null Energy Condition (NEC). Unusual properties of this hypothetical source make us return to the issues which had been seemingly clarified a long time ago but for sources that satisfy the standard energy conditions. The cosmological challenge has an impact on other areas of gravitational physics. It concerns the existence and properties of wormholes for which NEC violation is necessary. In black hole physics, the necessary conditions of regularity include the (marginal) validity of the NEC at the horizon [2], but the entire relationship between the properties

¹e-mail: kb20@yandex.ru

²e-mail: elizalde@ieec.uab.es

³e-mail: ozaslav@kharkov.ua

of matter and the near-horizon geometry remains unclear. In addition, NEC violation can play a significant role in the possible emergence of the so-called truly naked black holes (TNBHs) [3, 4], a class of objects in which infinite tidal accelerations in a freely falling reference frame is compatible with finiteness of the algebraic curvature invariants like the Kretschmann scalar. In cosmology, near-horizon phenomena are especially relevant in the context of the so-called Null Big Bang scenarios [5, 6] where the cosmological evolution itself begins with a horizon. Also, the possible importance of some of these models in generating dynamic, perturbatively small vacuum fluctuation contributions to the cosmological constant (within the dynamical Casimir effect [7] approach to this problem) will be considered, too. In this paper, we briefly review some recent results in this area. We will consider three different but related issues: Null Big Bang scenarios, possible black hole hair of matter characterized by macroscopic equations of state, and a relationship between two different classifications of near-horizon geometries according to their analyticity properties (hence extensibility beyond the horizon) and the properties of tidal forces acting on a freely falling body.

In this paper, for simplicity, we restrict ourselves to spherically symmetric space-times, though extension of the results to more general geometries would surely be of interest. More details can be found in our works [8–10].

2 Null Big Bang and its matter sources

We begin our considerations with spherically symmetric cosmological models characterized by the general Kantowski-Sachs (KS) metric

$$ds^2 = b^2 dt^2 - a^2 dx^2 - r^2(t) d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \quad (1)$$

and supported by a source with the stress-energy tensor

$$T_{\mu}^{\nu}(\text{tot}) = T_{\mu}^{\nu}(\text{vac}) + T_{\mu}^{\nu}(\text{matt}), \quad (2)$$

where

$$T_{\mu}^{\nu}(\text{vac}) = \text{diag}(\rho_v, \rho_v, -p_{v\perp}, -p_{v\perp}) \quad (3)$$

describes a “vacuum fluid” (defined by the condition $T_0^0(\text{vac}) = T_1^1(\text{vac})$ which guarantees invariance of $T_{\mu}^{\nu}(\text{vac})$ under any Lorentz boosts in the distinguished x -direction [11]) and

$$T_{\mu}^{\nu}(\text{matt}) = \text{diag}(\rho_m, -p_{mx}, -p_{m\perp}, -p_{m\perp}) \quad (4)$$

is the contribution of matter (anisotropic fluid) taken in the most general form compatible with the symmetry of the metric (1). We shall see that the properties of the system strongly depend on whether or not there is a “vacuum” admixture to such matter.

In what follows, it is helpful to use the so-called quasiglobal time coordinate, such that $b = a^{-1}$. The coordinate defined in this way, as well as its counterpart in static spherically-symmetric metrics, has two important advantages [12, 13]: (i) it always takes finite values $t = t_h$ at Killing horizons that separate static or cosmological regions of space-time from one another; (ii) near a horizon, the increment $t - t_h$ is a multiple (with a nonzero constant factor) of the corresponding increments of manifestly well-behaved Kruskal-type null coordinates, used for analytic continuation of the metric across the horizon. This condition implies the analyticity requirement for both metric functions $a^2(t)$ and $r^2(t)$ at $t = t_h$. Though, for our consideration, it is quite sufficient to require that these functions belong to class C^2 of smoothness.

With this coordinate gauge, two independent combinations of Einstein’s equations, chosen as $\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, read (the dot denotes d/dt)

$$\frac{2\ddot{r}}{r} a^2 = -8\pi(\rho_m + p_{mx}). \quad (5)$$

$$\frac{1}{r^2}(1 + \dot{r}^2 a^2 + 2a\dot{a}r\dot{r}) = 8\pi(\rho_m + \rho_v). \quad (6)$$

Assuming the absence of interaction between matter and vacuum, the conservation law $\nabla_\nu T^\nu_\mu = 0$ should hold for each of them separately. Taking the component with $\mu = 0$, we obtain

$$\dot{\rho}_m + \frac{\dot{a}}{a}(\rho_m + p_{mx}) + \frac{2\dot{r}}{r}(\rho_m + p_{m\perp}) = 0 \quad (7)$$

for matter and a similar equality for vacuum.

Let us assume $\rho_m \geq 0$ and consider different kinds of matter: “normal” matter that respects the NEC,

$$T_{\mu\nu}\xi^\mu\xi^\nu \geq 0, \quad \xi_\mu\xi^\mu = 0, \quad (8)$$

and “phantom” matter that violates it. Taking in (8) the null vector $\xi^\mu = (a, a^{-1}, 0, 0)$ we obtain the necessary conditions for NEC validity

$$\rho_m + p_{mx} \geq 0. \quad (9)$$

For normal matter, by definition, Eq. (9) holds, and consequently, according to Eq. (5), $\ddot{r} \leq 0$. So we can repeat the argument of [6]: let the system be expanding ($\dot{r} > 0$) at some t_1 . Then, either $r \rightarrow 0$ at some earlier instant $t_s < t_1$ (which means a curvature singularity) or the singularity is not reached, which can only happen due to a Killing horizon at some instant $t_h > t_s$. We have the following general result:

(i) *With any normal matter, regular cosmological evolution can only begin with a Killing horizon.*

Now, let us assume that there is a horizon at some $t = t_h$, so that, as $t \rightarrow t_h$, r remains finite while

$$a^2(t) \approx a_0(t - t_h)^n, \quad n \in \mathbb{N}, \quad (10)$$

where n is the order of the horizon. Then it immediately follows from (5) and the horizon regularity requirement (which implies analyticity of $r(t)$ and, in particular, finiteness of \ddot{r}) that

$$\rho_m + p_{mx} \rightarrow 0 \quad \text{as} \quad t \rightarrow t_h. \quad (11)$$

Furthermore, we can generically assume that near the horizon the pressure of our matter behaves as $p_{mx} \approx w\rho_m$, $w = \text{const}$. Then (assuming $|p_\perp|/\rho < \infty$) Eq. (7) implies the approximate equality

$$\rho_m \approx \text{const} \cdot a^{-(w+1)}, \quad (12)$$

near the horizon. This leads to the following two inferences:

(ii) *Non-interacting normal matter cannot exist in a KS cosmology with a horizon; thus it can only appear there due to interaction with the vacuum fluid.*

(iii) *Normal matter could only appear after a null big bang due to interaction with a sort of vacuum.*

This generalizes the conclusions made in [6] for KS cosmologies with dustlike matter.

As far as phantom matter is concerned, the presence of a Killing horizon is not necessary for obtaining a nonsingular cosmology. If such a horizon does exist, phantom matter can be present but with the restriction $w \leq -3$. Then, an analysis of Eqs. (5)–(7) near the horizon shows that we can have a simple or multiple horizon with $\rho_v(r_h) \neq 0$ or only a simple one with $\rho_v(r_h) = 0$. In such cases, a universe appearing in a Null Bang is initially contracting in the two spherical directions, $\dot{r} < 0$.

There is also a variant in which the Universe began its evolution infinitely long ago from an almost static state, which kind of evolution has been called “emergent universes” [14]. It should be pointed out here that, in the KS framework, unlike isotropic cosmologies, there exists a variant of nonsingular evolution in which one of the scale factors in the metric (1), namely, $a(t)$, vanishes as $\tau \rightarrow -\infty$ (where τ is the cosmological proper time, related to the quasiglobal time t by $d\tau = \int dt/a(t)$) while the other, $r(t)$, remains finite in the same limit, and both timelike and null geodesics starting from $\tau = -\infty$ are

complete. This is what can be called a *remote horizon* in the past, by analogy with remote horizons in static space-times mentioned in [9, 13].

We will illustrate this opportunity with two examples of such a generic behaviour as $\tau \rightarrow -\infty$:

$$\mathbf{A}: \quad a \approx a_0 e^{Ht} \sim 1/|\tau|, \quad r \approx r_0 + r_1 e^{Ht} \quad (13)$$

$$\mathbf{B}: \quad a \approx a_0 [t_0/(-t)]^q \sim |\tau|^{-q/(q+1)}, \quad r \approx r_0 + r_1 [t_0/(-t)]^{-s}, \quad (14)$$

where $a_0, r_0, r_1, H, t_0, s, q = \text{const} > 0$. Then, an analysis shows that in case A $w = -4$ and in case B $w = -3 - (s+2)/q < -3$. Also, in both cases, the conservation equation leads to the asymptotic structure of the vacuum stress tensor

$$T_{\mu}^{\nu}(\text{vac}) = \text{diag}(\rho_v, \rho_v, 0, 0). \quad (15)$$

Thus a combination of the Null Big Bang and emergent universe scenarios is possible but only under some special conditions: matter with $w \leq -3$ and a particular structure of the vacuum stress-energy tensor in the remote past.

In our reasoning, relying on the asymptotic behaviour of the density and pressure near the horizon, we did not assume any particular equation of state and even did not restrict the behaviour of the transverse pressure except for its regularity requirement. In this sense, our conclusions are model-independent. The fact that the very assumption of the existence of a cosmological horizon entails a number of rather general conclusions resembles, to some extent, the situation in black hole physics where the presence of the horizon greatly simplifies the description of the system and reduces the number of possibilities.

3 A black hole surrounded by matter

In the previous section, we dealt with cosmological evolution. Now, let us discuss the relationship between the properties of matter and the near-horizon geometry in static, spherically symmetric space-times. Such a problem arises in black hole physics. In real astrophysical conditions, black holes do not exist in empty space but are rather surrounded by some kind of matter which is either in equilibrium with the black hole or is falling on it. Meanwhile, the famous no-hair theorems (see, e.g., [2, 15] and references therein) are not directly applicable to such situations of evident astrophysical interest.

As before, we will rely on the horizon regularity condition, the Einstein equations and the conservation law for matter. The manner of reasoning is close to that of previous section. Instead of (1), we now have the metric

$$ds^2 = A(u)dt^2 - \frac{du^2}{A(u)} - r^2(u)(d\theta^2 + \sin^2\theta d\phi^2), \quad (16)$$

which is written using the quasiglobal radial coordinate u , similar to the quasiglobal time t of Section 2 and specified by the “gauge” condition $g_{00}g_{11} = -1$. We suppose that the vacuum fluid and matter have the stress-energy tensor (SET) given by (3) and (4), where p_{mx} (pressure of matter in the “longitudinal” direction in KS cosmology) is replaced by the radial pressure p_{mr} . Note that, in static, spherically symmetric space-times, examples of vacuum fluids are the cosmological constant ($p_{v\perp} = -\rho_v = \Lambda$), linear or nonlinear electric or magnetic fields in the radial direction ($p_{v\perp} = \rho_v$) and other forms which may be specified, e.g., by ρ_v as a function of r [11, 16–18].

Two independent combinations of Einstein’s equations, similar to (5) and (6), read (the prime means d/du)

$$G_0^0 - G_1^1 \equiv 2A \frac{r''}{r} = -8\pi(\rho_m + p_{mr}), \quad (17)$$

$$G_1^1 \equiv \frac{1}{r^2}[-1 + A'rr' + Ar'^2] = -8\pi(\rho_v - p_{mr}). \quad (18)$$

We again suppose that matter and the vacuum fluid do not interact with each other. Then the conservation law for matter reads

$$p_r' + \frac{2r'}{r}(p_{mr} - p_{m\perp}) + \frac{A'}{2A}(\rho_m + p_{mr}) = 0. \quad (19)$$

Now, assuming that there is a horizon at some $u = u_h$, a necessary condition of its regularity is that in its neighbourhood

$$A(u) \approx a_0(u - u_h)^n, \quad n \in \mathbb{N}, \quad (20)$$

where n is the order of the horizon. Another regularity condition is a smooth (at least C^2) behaviour of the other metric coefficient, $r^2(u)$.

One more assumption is that near the horizon the radial pressure of matter behaves as $p_{mr} \approx w\rho_m$, $w = \text{const}$. Then, on the basis of Eqs. (17)–(19) and the horizon regularity conditions, we can prove the following.

Theorem 1. *A spherically symmetric black hole can be in equilibrium with a static matter distribution with the SET (4) only if near the event horizon ($u \rightarrow u_h$, where u is the quasiglobal radial coordinate) either (i) $w \rightarrow -1$ (matter in this case has the form of a vacuum fluid) or (ii) $w \rightarrow -1/(1+2k)$, where $w \equiv p_r/\rho$ and k is a positive integer. In case (i), the horizon can be of any order n , and $\rho(u_h)$ is nonzero. In case (ii), the horizon is simple, and $\rho \sim (u - u_h)^k$.*

The generic case of such a non-vacuum hairy black hole is $k=1$, implying $w = -1/3$. In the case of an isotropic fluid, $p_r = p_\perp$, it corresponds to a distribution of disordered cosmic strings [19]. Since such strings are, in general, arbitrarily curved and may be closed, one can express the meaning of the theorem by the words “non-vacuum black holes can have curly hair”. Recall, however, that in general our w characterizes the radial pressure, while the transverse one is only restricted by the condition $|p_\perp|/\rho < \infty$.

Other values of k ($k = 2, 3$ etc.) represent special cases obtainable by fine-tuning the parameter w .

In the presence of vacuum matter with the SET (3), the following theorem holds:

Theorem 2. *A spherically-symmetric black hole can be in equilibrium with a non-interacting mixture of static non-vacuum matter with the SET (4) and vacuum matter with the SET (3) only if, near the event horizon ($u \rightarrow u_h$), $w \equiv p_r/\rho \rightarrow -n/(n+2k)$, where $n \in \mathbb{N}$ is the order of the horizon, $n \leq k \in \mathbb{N}$ and $\rho \sim (u - u_h)^k$.*

Thus a horizon of a static black hole can in general be surrounded by vacuum matter and matter with $w = -1/3$, which is true for any order of the horizon (i.e., including extremal and superextremal black holes) if $n = k$. There can also be configurations with $k > n$ and fine-tuned equations of state where $w = -n/(n+2k) > -1/3$. An arbitrarily small amount of other kinds of matter, normal or phantom, added to such a configuration, should break its static character by simply falling onto the horizon or maybe even by destroying the black hole. In other words, black holes may be hairy, or “dirty”, but the possible kinds of hair are rather special in the near-horizon region: normal (with $p_r \geq 0$) or phantom hair are completely excluded. In an equilibrium configuration, all “dirt” is washed away from the near-horizon region, leaving there only vacuumlike or modestly exotic, probably “curly” hair.

In particular, a static black hole cannot live inside a star of normal matter with nonnegative pressure unless there is an accretion region around the horizon or a layer of string and/or vacuum matter.

We did not discuss the behaviour of $p_{m\perp}$ and $p_{v\perp}$. In fact, these quantities are inessential for our reasoning. The latter is entirely local, restricted to the neighborhood of the horizon, and the results, which involve the single parameter $w = p_r/\rho|_{\text{horizon}}$, are in other respects model-independent. Meanwhile, a full analysis of specific systems would require the knowledge of the equation of state (including the properties of $p_{m\perp}$ and $p_{v\perp}$) and conditions on the metric in the whole space (e.g., the asymptotic flatness condition). Such an analysis depends on the model in an essential way and is beyond the scope of this paper. One can add that the equations of state well-behaved near the

horizon are often incompatible with reasonable conditions at infinity (see, e.g., the example of an exact solution with string fluid in [9]); it simply means that such matter does not extend to infinity and can only occupy a finite region around the horizon.

Our inferences are quite general and hold for all kinds of hair: for instance, in all known examples of black holes with scalar fields (see, e.g., [20] and references therein), the SETs near the horizon must satisfy the above conditions, which may be directly verified.

Also, our approach is relevant to semiclassical black holes in equilibrium with their Hawking radiation (the Hartle-Hawking state), whose SET essentially differs from that of a perfect fluid. Since the density of quantum fields is, in general, nonzero at the horizon (see Sec. 11 of the textbook [2] for details), the regularity condition (11) (with t replaced by r) tells us that such quantum radiation should behave near the horizon like a vacuum fluid. Our results show that a black hole can be in equilibrium with a mixture of Hawking radiation and some kinds of classical matter with $-1 < w < 0$ (including the important case of a Pascal perfect fluid with $p_r = p_\perp$). Possible effects of this circumstance for semiclassical black holes need a further study. Moreover, large enough black holes, for which the Hawking radiation may be neglected, can be in equilibrium with classical matter alone, also including the case of a perfect fluid.

It would be of interest to generalize our results to nonspherical and rotating distributions of matter.

4 Truly naked horizons and their sources

In the previous two sections, the restriction on possible matter sources supporting geometries with Killing horizons essentially relied on the horizon regularity condition, which essentially meant analyticity. Meanwhile, the notion of regularity is by itself not as obvious as one could think. In particular, it turns out that there exist such horizons that all scalars composed algebraically from the components of the curvature tensor are finite there but some separate curvature components (responsible for the transverse tidal forces) enormously grow when approaching the horizon [10, 13, 21–23]. From a mathematical viewpoint, such cases represent interesting examples of so-called nonscalar singularities [24]. This makes especially important a careful analysis of the metric near the surfaces which can be called candidate horizons and a comparison between the properties of tidal forces on such surfaces and the conditions under which the metric can be extended beyond them.

Let the metric in the Schwarzschild-like coordinates be written as

$$ds^2 = e^{2\gamma} dt^2 - e^{2\alpha} dr^2 - r^2 d\Omega^2. \quad (21)$$

Let us assume that near a candidate horizon $\mathbb{H} : r = r_h$ (where, by definition, $e^\gamma \rightarrow 0$) the metric coefficients behave as follows:

$$e^{2\gamma} \sim (r - r_h)^q, \quad e^{2\alpha} \sim (r - r_h)^p, \quad (22)$$

with $p > 0$ and $q > 0$. As follows from the geodesic deviation equations, the tidal forces experienced by bodies in the gravitational field are conveniently characterized by the combination of components of the curvature tensor $Z := R^{12}_{12} - R^{02}_{02}$ in the static reference frame and by $\bar{Z} = Z e^{-2\gamma}$ in a freely falling reference frame near \mathbb{H} . These quantities have been used in [4] to distinguish usual ($Z = 0 = \bar{Z}$), naked ($Z = 0, \bar{Z} \neq 0$ is finite) and truly naked ($Z = 0, \bar{Z} = \infty$) horizons. In all cases we consider surfaces \mathbb{H} at which all algebraic curvature invariants are finite, and this is so under the condition

$$p \geq 2 \quad \text{or} \quad 2 > p \geq 1, \quad p + q = 2. \quad (23)$$

The comparison is carried out by rewriting the metric (21) in terms of the quasiglobal coordinate u [Eq. (16)] and imposing the requirement that the metric coefficients $A(u)$ and $r^2(u)$ should be analytic at \mathbb{H} , where $A(u) = e^{2\gamma(r)} = 0$. In particular, we obtain the condition

$$q(n - 2) = n(p - 2). \quad (24)$$

which selects a sequence of lines in the (p, q) plane, intersecting at the point $(-2, 0)$.

Table 1: Horizon types according to the properties of tidal forces and regularity (extensibility) of the metric; $n \in \mathbb{N}$ is the order of the horizon.

No.	p, q	type by tidal forces	regularity
1	$p = q = 1$	usual or naked	regular, $n = 1$
2	$1 < p < 3/2, q = 2 - p$	truly naked	singular
3	$p = 3/2, q = 1/2$	naked	regular, $n = 1$
4	$3/2 < p < 2, q = 2 - p$	usual	regular, $n = 1$
5	$p \geq 2, q > p$	truly naked	singular
6	$p \geq 2, q = p$	usual or naked	regular if $p = q = n$, otherwise singular
7	$p \geq 2, p - 1 < q < p$	truly naked	singular
8	$p \geq 2, q = p - 1$	naked	regular if $p = 1 + n/2$, otherwise singular
9	$p \geq 2, p - 2 < q < p - 1$	usual	regular if (24) holds, $n \in \mathbb{N}$, otherwise singular
10	$p \geq 2, q \leq p - 2$	usual	remote horizon

The results of such a comparison are presented in Table 1 [10].

We see from the table that all truly naked horizons are, in fact, singularities since the metric cannot be extended beyond them; moreover, some naked and even usual horizons turn out to be singular.

As to possible source of gravity leading to different types of horizons, the situation turns out to be the following. If we consider arbitrary one-component matter with $p_r/\rho = w = \text{const} \neq -1$ (at least near the surface \mathbb{H}), the only possible solutions correspond to a simple horizon, such that $A(u) \sim u - u_h$, and, provided $w = -1/(1+2k)$ where k is a positive integer, we obtain regular solutions in full agreement with Section 3. Solutions with truly naked horizons are not obtained.

If, however, we consider a mixture of the two kinds of matter described by (4) and (3), there appear solutions containing matter with $0 > w > -1$ and $\rho \sim A^{-(w+1)/(2w)}$. Furthermore, if we turn to the curvature coordinates, we obtain, for $p \neq q$, the relation $w = -q/(q+2p-2)$, and it appears that $\rho > 0$ for $q > p$ and $\rho < 0$ for $q < p$. Thus, any p and q satisfying the condition (23) are admissible, except for those with $p = q$. In the latter case, solutions can also exist, with w satisfying the requirement $w > -p/(3p-2)$. All kinds of solutions mentioned in Table I are possible, and the values of w cover the whole range from 0 to -1 . This is related to the underdetermined nature of the system since the function $\rho_{(\text{vac})}(u)$ remains arbitrary.

If we put $\rho_v = \Lambda/(8\pi) = \text{const}$, thus specifying the vacuum as a cosmological constant, the Einstein equations relate the exponents p and q characterizing the metric to the matter parameter w . Namely, we have either (i) $p = q = 1$ (a simple regular horizon) and $w = -1/(1+2k)$ (as described in Section 3 and [9]) or (ii) $p = 2, q = -2w/(w+1), w \neq -1/2$. The parameter w can take any value in the range $(-1, 0)$ except $-1/2$.

5 Conclusion

In a model-independent way, we have established the correspondence between the equation of state (in terms of the parameter $w = p_r/\rho$) and the type of horizon both for cosmological scenarios and for black holes. We found the interval of w for which regular or (truly) naked horizons occur. Certain discrete values of w characterize possible “hair” around a regular black hole horizon. Thus we have used a unified approach to so seemingly different physical objects and phenomena as Null Big Bang, the hair properties of black holes and (truly) naked black holes.

This consideration, along with [6, 8], suggests an interesting type of cosmological scenarios, with such stages as (i) a static or stationary core, (ii) a de Sitter-like horizon, (iii) particle creation and isotropization, (iv) a hot stage and further on according to the Standard Model.

In the context of the early Universe, in addition to particle creation, it would be of interest to take

into account one more quantum phenomenon, the ordinary [25] and the dynamical [26] Casimir effects related to the nontrivial topology of KS models, e.g., in the manner of Refs. [27, 28], and its possible influence on the structure of singularities like those discussed in this paper. It has been argued that Casimir considerations can play no role in trying to solve the problem of the cosmological constant, in its hard form. This seems actually to be true, but provided a drastic suppression of the main contributions to the same for some particular topology could be proven to happen in some model, then additional, sort of perturbative contributions coming from some adjustments in the topology or the evolution of our universe could provide a clue to solve the issue of its value being so small. It is in this context that Casimir-like calculations as mentioned could be of importance. At the very least, proving that these additional contributions are of the same order of magnitude as the observed value of the universe acceleration is already a first step, that has been undertaken in some specific cases [29].

It would also be of interest to relate the origin of singularities in KS cosmology subject to quantum effects with their effective 2D description, in the manner of [30], where quantum-corrected KS cosmologies were investigated. The effective 2D description makes the presentation qualitatively easier and may reveal a fundamental structure behind singularities, related to quantum effects.

As to singular horizons in KS cosmology as discussed in this paper (at least simple ones), they can be considered as examples of the so-called finite-time singularities. Four types of such singularities are known and classified for isotropic FRW models in [31]. Among them, the Big Rip (or type I singularity) is the most well-known and is widely discussed in connection with different models of dark energy. It is clear that in KS models, where we have two scale factors, such singularities may occur and their properties should be more diverse, and the corresponding classification should be naturally extended as compared with the one-scale-factor FRW cosmology. Such an extended classification, which should also apply to static, spherically symmetric analogues of KS cosmologies as well as to other, more complicated anisotropic cosmologies, is of significant interest. We hope to present a detailed description of such singularities in our future publications.

It is a pleasure for us to dedicate this paper to Sergei Odintsov on the occasion of his 50th birthday.

Acknowledgement

E.E. has been supported in part by MEC (Spain), project FIS2006-02842, and by AGAUR (Generalitat de Catalunya), contract 2005SGR-00790. K.B. has been supported by the Russian Basic Research Foundation grant No. 07-02-13624-ofi-ts and by a grant of People Friendship University of Russia (NPK MU). The work of O.Z. was supported by European Science Foundation, Short Visits and Exchange Programme, grant # 2536.

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